# Comment on "A note on the scaling relations for opening mode fractures in rock" by C.H. Scholz

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## ARTICLE INFO

Olson (2003) was written to propose an alternative fracture mechanics explanation for fracture aperture vs. length relationships found in field data. The premise was that under certain loading histories, brittle fractures should be expected to propagate at roughly *constant stress intensity factor* (equal to either fracture toughness, $K_{Ic}$, or to a subcritical threshold value, $K_{Ic}*$). This hypothesis predicts a square-root relationship between fracture aperture and length. Other authors have proposed models for linear scaling of fracture aperture to length presuming crack propagation at *constant driving stress* (Pollard and Segall, 1987; Renshaw and Park, 1997), a condition traditionally considered unstable for opening mode cracks based on Griffith's energy balance approach to crack propagation (Lawn and Wilshaw, 1975; Segall, 1984). Olson (2003) used a published opening mode vein dataset of Vermilye and Scholz (1995) and a published igneous dike dataset of Delaney and Pollard (1981) to corroborate the theoretical arguments for square root scaling. Subsequent to the publication of Olson (2003), Schultz et al. (2008a, 2008b) contributed additional supporting empirical data to the square root scaling hypothesis.

Scholz (2010) revisited the topic of aperture vs. length scaling as discussed in Olson (2003), arguing three major points –

1) constant stress intensity factor propagation hypothesized in Olson (2003) is unlikely because fracture toughness should increase instead as the square root of fracture length ($K_{Ic} \propto \sqrt{L}$),
2) some of the square root scaling data presented by Olson (2003), Schultz et al. (2008a, 2008b) instead indicate linear scaling, and
3) some of the data presented by Olson (2003), Schultz et al. (2008a, 2008b) are irrelevant because they consist of en echelon arrays of dikes which are mechanically interacting and cannot be interpreted using a simple idealized model.

In this comment, we would like to counter those assertions.

Firstly, Scholz (2010) proposes that fracture toughness scales as the square root of fracture length in rock ($K_{Ic} \propto \sqrt{L}$). A toughness dependence on fracture length of this form is not supported by published laboratory experiments. Experimental data from Labuz et al. (1985) show a square-root scale increase in fracture toughness for shorter cracks ranging from 10 to 50 mm in length for coarse-grained granite, but the results of Labuz et al. (1987) for the same granite show no such relationship for longer cracks ranging from 80 to 160 mm in length - for these cases toughness is roughly constant. Weisinger et al. (1980) saw only a weak dependence of toughness on fracture length ($\sim L^{0.1}$) in Nevada Tuff. Kobayashi et al. (1986) and Rose (1986) both argue that once a crack reaches a certain size (on the order of 100 mm), such that the process zone is small relative to the overall crack length, toughness becomes independent of crack length. Finally, a theoretical study (Sato and Hashida, 2006) accounting for cohesive effects at the crack tip predicted that toughness should scale not with crack length but roughly with the ratio of process zone radius to crack length ($r_p/L$). If this model is true, then a process zone that scales linearly with fracture length as Scholz argues would imply a constant ratio, $r_p/L$, and a constant $K_{Ic}$.

Thus, we interpret that, at best, laboratory experiments are supportive of the premise that fracture toughness is relatively constant over outcrop length-scales encompassing growth of a single set of opening mode fractures (joints or veins), or, at worst, the laboratory data are inconclusive. However, as Scholz (2010) points out, Olson (2003) did acknowledge that if we jump in scale from meter long joints and veins to kilometer-scale dikes, fracture toughness has been estimated to be much higher (e.g., Delaney and Pollard, 1981). Nonetheless, there is a body of literature that demonstrates that fracture toughness for igneous dikes such as Ship Rock and other examples is not simply a fracture length dependent property, but the effective toughness for dike

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propagation is influenced by in-situ stress magnitude, rock tensile strength, scale of the rock mass, and temperature (e.g., Pollard and Segall, 1987; Rubin, 1993; Bunger, 2008; Schultz et al., 2008a). As a result, fracture toughness cannot be a simple function of fracture length (such as $\sqrt{L}$) as Scholz (2010) suggests.

Scholz' critique of the statistical interpretation of aperture to length scaling in Olson (2003) follows largely from his presumption that toughness scales with square root of fracture length. We argue the statistics should be evaluated independently and the results used to assess the veracity of the theoretical assertions (whether fractures propagate with constant stress intensity factor or constant driving stress). In our analyses, presented in Olson (2003), Schultz et al. (2008a, 2008b), we find that square root scaling is the best description of the observations when treating each dataset independently (these data are reprinted in Fig. 1 of Scholz, 2010). This approach is consistent with the statistical work of Clark and Cox (1996), who showed that a single regression line was not appropriate to describe multiple fault datasets, but rather that each individual dataset could and should be fit by its own regression. The fact that each dataset points to a different toughness value operating at the time and locality for that fracture set, as plotted in Fig. 2 of Scholz (2010), does not refute the hypothesis of constant stress intensity factor propagation for each fracture set.

Finally, Scholz discounts the use of datasets comprised of segmented structures, such as the Ship Rock dike and the dike array from Ethiopia measured and reported by Schultz et al. (2008a). Olson (2003) demonstrated that mechanical interaction between fracture segments can account for an aperture versus length slope that is reduced from the slope associated with non-interacting segments. Variations in segment spacing and overlap are well known to contribute to scatter about the unit slope for faults, and we interpret the scatter in slopes of datasets comprised of multiple interacting opening mode crack segments as being related to the average segment geometry and mechanical interaction. The dike datasets are clearly consistent with $\sqrt{L}$ scaling of mechanically interacting en echelon dike segments.

Overall, we maintain that aperture vs length scaling datasets are an excellent way to evaluate the physics of crack propagation in the earth. We disagree with Scholz that opening mode fractures must scale in the same manner as faults (consistent with the distinction in propagation mechanics between opening mode fractures and faults noted by Scholz, 2002, p. 116) and feel the best interpretation for opening mode fractures is square root scaling, but we recognize there is still room for debate as the number of good datasets available remains relatively small. We commend Scholz and his coworkers for being providers of some of the best datasets available (such as the Vermilye and Scholz, 1995, observations), and look forward to continued progress on this subject.

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